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NOTES

CLASS:- MBA 3rd SEM

SUBJECT: OPERATION RESEARCH

Unit-1

Operational Research (OR) is an interdisciplinary field that combines analytical methods, mathematical models, and data analysis to make better decisions in complex systems. Also known as Operations Research or Management Science, OR has its roots in the military and industrial settings of the mid-20th century.

Meaning:

Operational Research involves using advanced analytical techniques to optimize decision-making processes in various fields, such as business, healthcare, finance, logistics, and more. It aims to identify the most effective solutions to complex problems, often involving trade-offs between competing objectives.

Origin:

The term "Operational Research" was first used in the UK during World War II to describe the application of scientific methods to military operations. After the war, OR expanded to other fields, including industry and management.

Scope:

The scope of Operational Research is broad and encompasses various aspects of decision-making, including:

1. Problem-solving
2. Decision analysis
3. Risk analysis
4. Optimization
5. Simulation
6. Modeling
7. Data analysis
8. Performance measurement

Role in Managerial Decision Making:
Operational Research plays a vital role in managerial decision-making by providing a structured approach to analyzing complex problems and making informed decisions. OR helps managers:
1. Identify and formulate problems
2. Develop and evaluate alternatives
3. Choose the best course of action

4. Implement and monitor solutions
5. Improve communication and collaboration

By applying OR techniques, managers can make more informed, data-driven decisions that optimize resources, minimize costs, and maximize efficiency.

Linear Programming (LP) is a method used to optimize a linear objective function, subject to a set of linear constraints. It is a powerful tool for decision-making and resource allocation in various fields, including operations research, management science, and economics.

Meaning:

Linear Programming involves finding the best outcome (maximum or minimum) of a linear objective function, subject to a set of linear constraints. The objective function represents the goal to be achieved, while the constraints represent the limitations and requirements of the problem.

Scope:

The scope of Linear Programming includes:
1. Resource allocation
2. Production planning
3. Scheduling
4. Transportation
5. Inventory control
6. Financial planning
7. Network optimization

Limitations:
1. Linearity assumption: LP assumes a linear relationship between variables, which may not always hold true.
2. Simplistic modeling: LP models can be oversimplified, failing to capture complex relationships and nuances.
3. Scalability: LP can become computationally expensive and difficult to solve for large, complex problems.
4. Data requirements: LP requires accurate and reliable data, which may be difficult to obtain.
5. Interpretation: LP solutions require careful interpretation and sensitivity analysis to ensure practical applicability.
6. Assumption of certainty: LP assumes certainty in the data and parameters, which may not always be the case.
7. Limited flexibility: LP models can be inflexible and difficult to adjust to changing circumstances.

Despite these limitations, Linear Programming remains a powerful tool for optimization and decision-making, widely used in various fields.
Here are some common formulations of industrial and business problems as linear programming problems:

1. Production Planning:

- Maximize profit = Σ (selling price - production cost) x quantity produced
- Subject to: production capacity, material availability, labor constraints

2. Resource Allocation:

- Maximize efficiency = Σ (resource utilization x resource cost)
- Subject to: resource availability, demand constraints

3. Inventory Control:

- Minimize cost = Σ (holding cost x inventory level) + ordering cost
- Subject to: inventory capacity, demand constraints

4. Transportation:

- Minimize cost = Σ (transportation cost x distance)
- Subject to: supply and demand constraints, route constraints

5. Scheduling:

- Maximize efficiency = Σ (machine utilization x machine cost)
- Subject to: machine availability, labor constraints, deadline constraints

6. Financial Planning:

- Maximize return = Σ (investment return x investment amount)
- Subject to: budget constraints, risk constraints

7. Supply Chain Optimization:

- Minimize cost = Σ (production cost + transportation cost + inventory cost)
- Subject to: supply and demand constraints, route constraints

8. Quality Control:

- Maximize quality = Σ (quality metric x production quantity)
- Subject to: production capacity, material constraints

9. Energy Management:

- Minimize cost = Σ (energy consumption x energy cost)
- Subject to: energy availability, demand constraints

10. Portfolio Optimization:

- Maximize return = Σ (investment return x investment amount)

These are just a few examples of how industrial and business problems can be formulated as linear programming problems. The specific formulation will depend on the details of the problem and the goals of the organization.
Here are the general steps for solving linear programming problems using graphical and simplex methods:

Graphical Method:

1. Plot the constraints on a graph.

2. Identify the feasible region.

3. Plot the objective function.

4. Find the optimal solution by graphically identifying the point where the objective function intersects the feasible region.

Simplex Method:

1. Convert the linear programming problem to standard form.

2. Create a simplex tableau.

3. Perform pivoting operations to transform the tableau into a form where the optimal solution is easily identified.

4. Read off the optimal solution from the **final tableau.**

Simplex Algorithm:

1. Initialize the tableau with the initial basic feasible solution (BFS).

2. Check if the current BFS is optimal. If yes, stop.

3. Select a non-basic variable to enter the basis.

4. Select a basic variable to leave the basis.

5. Perform pivoting to update the tableau.

6. Repeat steps 2-5 until the optimal solution is reached.

***Example**:*

Maximize: $2x + 3y$

Subject to:

$x + y \leq 4$

$2x - y \leq 3$

$x, y \geq 0$

Using the graphical method, we can plot the constraints and objective function on a graph and identify the optimal solution as x = 2, y = 2.

Using the simplex method, we can convert the problem to standard form, create a simplex tableau, and perform pivoting operations to find the optimal solution as x = 2, y = 2.

Note: The simplex method is a more systematic and efficient approach for solving linear programming problems, especially for larger problems. The graphical method is useful for small problems and for visualizing the solution.

Degeneracy and duality are two important concepts in linear programming.

Degeneracy:

A linear programming problem is said to be degenerate if there are multiple optimal solutions or if the optimal solution is not unique.
- In other words, degeneracy occurs when the problem has multiple extreme points with the same objective function value.
- Degeneracy can lead to difficulties in solving the problem and interpreting the results.

Duality:

- Duality is a fundamental concept in linear programming that relates to the relationship between the primal problem and its dual problem.

- The primal problem is the original linear programming problem, while the dual problem is derived from the primal problem by transposing the coefficient matrix and changing the optimization direction (maximization to minimization or vice versa).

- The dual problem provides a lower bound on the optimal value of the primal problem, and the optimal value of the dual problem is equal to the optimal value of the primal problem.

- Duality is useful for solving linear programming problems, as it provides an **alternative formulation that can be easier to solve.**

Types of Duality:

- Weak Duality: The optimal value of the dual problem is less than or equal to the optimal value of the primal problem.
- Strong Duality: The optimal value of the dual problem is equal to the optimal value of the primal problem.

Importance of Duality:

- Provides a deeper understanding of the linear programming problem

- Helps in solving linear programming problems

- Useful in sensitivity analysis and post-optimality analysis

- Provides a connection between linear programming and other areas of mathematics, such as game theory and convex analysis.

Unit-2

Transportation problems are a type of linear programming problem that deals with the transportation of goods from one place to another. There are two types of transportation problems: balanced and unbalanced.

Balanced Transportation Problem:

- A transportation problem is said to be balanced if the total supply equals the total demand.
- In other words, the sum of the supplies from all sources equals the sum of the demands at all destinations.
- A balanced transportation problem can be solved using the transportation algorithm.

Unbalanced Transportation Problem:

- An unbalanced transportation problem occurs when the total supply does not equal the total demand.
- This can happen when there is excess supply or excess demand.
- An unbalanced transportation problem can be converted into a balanced problem by adding a dummy source or destination to absorb the excess supply or demand.

Solving Unbalanced Transportation Problems:

- Add a dummy source or destination to absorb excess supply or demand.
- Assign a zero transportation cost to the dummy source or destination.
- Solve the problem using the transportation algorithm.

Important **Considerations:**

- Identify whether the problem is balanced or unbalanced.
- Convert unbalanced problems into balanced problems by adding a dummy source or destination.
- Use the transportation algorithm to solve balanced transportation problems.
- Consider the possibility of degeneracy and use techniques like perturbation or preprocessing to avoid it.

Note: Transportation problems can be solved using various methods, including the transportation algorithm, linear programming relaxation, and network simplex method.

The Northwest Corner Method is a heuristic used to find an initial basic feasible solution to a transportation problem. Here's how to apply it:

1. Start with the top-left cell (i.e., the cell in the first row and first column).

2. Allocate the maximum possible value to this cell, which is the minimum of the supply and demand.

3. Move to the next cell in the row (i.e., the cell to the right) and allocate the maximum possible value to it, which is the minimum of the remaining supply and demand.

4. Repeat step 3 for each row, moving from left to right.

5. Once all rows have been processed, move to the next column and repeat steps 2-4.

6. Continue this process until all cells have been allocated a value.

The resulting solution is an initial basic feasible solution, which can then be improved upon using the transportation algorithm.

Example:

Suppose we have a transportation problem with the following supply and demand:

	A	B	C	Supply
1	10	20	30	100
2	40	10	20	150
3	20	30	40	120
Demand	80	120	100	

Using the Northwest Corner Method, we get:

	A	B	C	Supply
1	10	20	0	100
2	40	0	20	150
3	20	30	40	120
Demand	80	120	100	

Unit-3

This is the initial basic feasible solution obtained by the Northwest Corner Method.

The Least Cost Entry Method is a method used to find an initial basic feasible solution to a transportation problem. It involves assigning values to the cells of the transportation table in a way that minimizes the total transportation cost.

Here are the steps to apply the Least Cost Entry Method:

1. Identify the cell with the lowest transportation cost (i.e., the cell with the smallest cost per unit).

2. Allocate the maximum possible value to this cell, which is the minimum of the supply and demand.

3. Make sure that the row and column totals are not exceeded.

4. If there are multiple cells with the same lowest transportation cost, choose the one with the highest demand.

5. Repeat steps 1-4 until all cells have been allocated a value.

The resulting solution is an initial basic feasible solution, which can then be improved upon using the transportation algorithm.

Example:

Suppose we have a transportation problem with the following supply and demand:

	A	**B**	**C**	**Supply**
1	**10**	**20**	**30**	**100**
2	**40**	**10**	**20**	**150**
3	**20**	**30**	**40**	**120**
Demand	**80**	**120**	**100**	

The **transportation costs per unit are:**

	A	**B**	**C**
1	2	3	4
2	3	2	1
3	4	3	2

Using the Least Cost Entry Method, we get:

	A	B	C	Supply
1	10	20	0	100
2	40	0	20	150
3	20	30	40	120
Demand	80	120	100	

This is the initial basic feasible solution obtained by the Least Cost Entry Method.

Vogel's Approximation Method (VAM) is a heuristic used to find an initial basic feasible solution to a transportation problem. It is based on the idea of assigning values to the cells of the transportation table in a way that minimizes the total transportation cost.

Here are the steps to apply VAM:

1. Calculate the penalty for each cell as the difference between the supply and demand.

2. Identify the cell with the highest penalty.

3. Allocate the maximum possible value to this cell, which is the minimum of the supply and demand.

4. Update the row and column totals.

5. Repeat steps 2-4 until all cells have been allocated a value.

VAM is similar to the Least Cost Entry Method, but it uses penalties instead of transportation costs to make assignments.

Example:

Suppose we have a transportation problem with the following supply and demand:

	A	B	C	Supply
1	10	20	30	100
2	40	10	20	150
3	20	30	40	120
Demand	80	120	100	

The penalties are calculated as:

	A	B	C
1	10-20=-10	20-10=10	30-20=10
2	40-10=30	10-20=-10	20-30=-10
3	20-30=-10	30-20=10	40-30=10

Using VAM, we get:

	A	B	C	Supply
1	10	20	0	100
2	40	0	20	150
3	20	30	40	120
Demand	80	120	100	

This is the initial basic feasible solution obtained by VAM.

The Modi and Stepping Stone methods are two popular techniques used to find the optimal solution to transportation problems.

Modi Method:

1. Find an initial basic feasible solution using any method (e.g., Northwest Corner, Least Cost, or Vogel's Approximation).
2. Calculate the net evaluations (NE) for each cell:
   - NE = Transportation cost - Dual variable
3. Identify the cell with the most negative NE (called the "incoming cell").
4. Draw a loop starting from the incoming cell, passing through the basic cells, and returning to the incoming cell.
5. Calculate the loop's net evaluation (NE-loop).
6. If NE-loop ≥ 0, the current solution is optimal. Otherwise, proceed to step 7.

7. Update the dual variables and basic variables accordingly

Stepping Stone Method:

1. Find an initial basic feasible solution using any method.
2. Create a "stepping stone" table with the following columns:
   - Cell
   - Transportation cost
   - Dual variable
   - Net evaluation (NE)
3. Identify the cell with the most negative NE (called the "incoming cell").
4. Draw a loop starting from the incoming cell, passing through the basic cells, and returning to the incoming cell.
5. Calculate the loop's NE-loop.
6. If NE-loop ≥ 0, the current solution is optimal. Otherwise, proceed to step 7.
7. Update the dual variables and basic variables accordingly.
8. Repeat steps 3-7 until the optimal solution is reached.

Both methods are used to improve upon an initial basic feasible solution and find the optimal solution to a transportation problem. The Modi method uses a more straightforward approach, while the Stepping Stone method uses a more visual approach with the stepping stone table.

Degeneracy in transportation problems refers to a situation where multiple cells in **the transportation table have the same minimum cost, making it difficult to determine which cell to choose for transportation. This can lead to multiple optimal solutions or an infinite number of solutions.**

Types of Degeneracy:

1. *Cell Degeneracy*: When multiple cells have the same minimum cost.

2. *Row Degeneracy*: When all cells in a row have the same minimum cost.

3. *Column Degeneracy*: When all cells in a column have the same minimum cost.

4. *Global Degeneracy*: When all cells in the transportation table have the same **minimum cost.**

Effects of Degeneracy:

1. *Multiple Optimal Solutions*: Degeneracy can lead to multiple optimal solutions, making it difficult to choose the best one.

2. *Infinite Number of Solutions*: In some cases, degeneracy can result in an infinite number of solutions.

3. *Difficulty in Finding the Optimal Solution*: Degeneracy can make it challenging to find the optimal solution using traditional methods.

Methods to Resolve Degeneracy:

1. *Perturbation*: Add a small random value to the costs to break the degeneracy.
2. *Preprocessing*: Transform the transportation table to reduce degeneracy.
3. *Specialized Algorithms*: Use algorithms designed specifically for degenerate transportation problems.
4. *Heuristics*: Use heuristic methods to find a good, but not necessarily optimal, solution.

It's important to note that degeneracy can be a challenging issue in transportation problems, and resolving it may require specialized techniques or algorithms.
The Assignment Problem and the Traveling Salesman Problem are both classical problems in Operations Research and Computer Science.

Assignment Problem:

- Definition: The Assignment Problem is a problem where we have a set of tasks and a set of agents, and we want to assign each task to exactly one agent in such a way that the total cost or time is minimized.
- Example: Assigning jobs to machines, assigning students to projects, etc.
- Solution methods: Hungarian Algorithm, Linear Programming Relaxation, etc.

Traveling Salesman Problem (TSP):

- Definition: The TSP is a problem where we have a set of cities and we want to find the shortest possible tour that visits each city exactly once and returns to the starting city.

- Example: A salesman wants to visit a set of cities and return home, minimizing the total distance traveled.

- Solution methods: Exact algorithms (e.g., Concorde), Heuristics (e.g., Nearest Neighbor, 2-opt), Metaheuristics (e.g., Simulated Annealing, Genetic Algorithms).

Both problems are NP-hard, meaning that the running time of traditional algorithms increases exponentially with the size of the problem instance. However, there are many approximation algorithms and heuristics that can be used to solve these problems in reasonable time.

Some key differences between the two problems are:

- The Assignment Problem is a minimization problem, while TSP is a minimization problem with an additional constraint (visiting each city exactly once).

- The Assignment Problem has a linear objective function, while TSP has a non-linear objective function (distance between cities).

1. Unbalanced Problem: When the number of tasks and agents is not equal, the problem is said to be unbalanced.
2. Maximization Objectives: Instead of minimizing the total cost or time, the objective is to maximize the total profit or utility.
3. Multiple Optimal Solutions: In some cases, there may be multiple optimal solutions with the same minimum cost or maximum profit.
4. Degeneracy: When multiple cells in the assignment table have the same minimum cost, the problem is said to be degenerate.
5. Infeasibility: When it is not possible to assign all tasks to agents, the problem is said to be infeasible.
6. Bottleneck Assignments: When a single agent is assigned multiple tasks, creating a bottleneck.
7. Multiple Tasks per Agent: When agents can perform multiple tasks simultaneously.
8. Task Dependencies: When tasks have dependencies, and the assignment of one task affects the assignment of another.
9. Agent Skills: When agents have different skills, and tasks require specific skills.
10. Time Windows: When tasks have time windows, and agents must complete tasks within those windows.

These special cases require modifications to the standard assignment algorithms or the use of specialized algorithms to solve.

PERT (Program Evaluation and Review Technique) and CPM (Critical Path Method) are both project management techniques used for planning, scheduling, and controlling projects. The main differences between PERT and CPM are:

1. Approach:
   - PERT: Focuses on the uncertainty of task durations and uses probabilistic estimates.
   - CPM: Uses deterministic estimates and focuses on the critical path activities.
2. Task Duration Estimates:
   - PERT: Uses three-point estimates (optimistic, most likely, and pessimistic) to account for uncertainty.
   - CPM: Uses single-point estimates (fixed duration).
3. Network Diagram:
   - PERT: Uses an activity-on-arrow (AOA) diagram.
   - CPM: Uses an activity-on-node (AON) diagram.
4. Critical Path:
   - PERT: Identifies multiple critical paths and near-critical paths.
   - CPM: Identifies a single critical path.
5. Scheduling:
   - PERT: Uses a simulation approach to schedule activities.
   - CPM: Uses a deterministic approach to schedule activities.
6. Resource Allocation:
   - PERT: Does not consider resource allocation explicitly.
   - CPM: Explicitly considers resource allocation and leveling.
7. Objective:
   - PERT: Focuses on completing the project within a specified time frame.
   - CPM: Focuses on minimizing the project duration.

In summary, PERT is more suited for projects with uncertain task durations, while CPM is better suited for projects with fixed task durations and a focus on resource allocation.

Network construction refers to the process of building a project network diagram, which is a visual representation of the project's activities, dependencies, and relationships. Here are the general steps involved in network construction:

Unit-4

Activity list

1. Identify the project activities: List all the activities that need to be performed to complete the project.
2. Determine the dependencies: Identify the dependencies between activities, including predecessor and successor relationships.
3. Draw the network diagram: Use a node (or box) to represent each activity and an arrow to represent the dependencies.
4. Add activity details: Include relevant information such as activity descriptions, durations, and resource requirements.
5. Check for errors: Verify that the network diagram is accurate, complete, and free of errors.

Some popular network construction techniques include:

1. Activity-on-node (AON)
2. Activity-on-arrow (AOA)
3. Precedence diagramming method (PDM)

Network construction is a critical step in project management, as it helps to:

1. Visualize the project scope and activities
2. Identify dependencies and relationships
3. Develop a project schedule
4. Assign resources and estimate costs
5. Identify potential risks and bottlenecks

By constructing a comprehensive project network diagram, project managers can better plan, coordinate, and control their projects.

EST (Early Start Time) is a metric used in project management to calculate the earliest possible start time of an activity. Here's how to calculate EST:

EST = max (EF predecessor, ES predecessor) + 1

Where:

- EF predecessor is the early finish time of the predecessor activity

- ES predecessor is the early start time of the predecessor activity

This calculation ensures that the activity starts after its predecessor has finished, or if there are multiple predecessors, it starts after the latest finish time of all predecessors.

For example, if an activity has two predecessors with EF times of 10 and 12, and ES times of 8 and 9 respectively, the EST would be:

EST = max (12, 9) + 1 = 13

This means that the activity can start at time 13, which is after the latest finish time of its predecessors.

Note that EST is used in conjunction with other metrics like EFT (Early Finish Time), LST (Late Start Time), and LFT (Late Finish Time) to calculate the float and slack in a project schedule.

EFT (Early Finish Time) is a metric used in project management to calculate the earliest possible finish time of an activity. Here's how to calculate EFT:

EFT = EST + Duration

Where:

- EST (Early Start Time) is the earliest possible start time of the activity

- Duration is the estimated time required to complete the activity

For example, if an activity has an EST of 10 and a duration of 5, the EFT would be:

EFT = 10 + 5 = 15

This means that the activity can finish at time 15, which is the earliest possible finish time.

Note that EFT is used in conjunction with other metrics like EST, LST (Late Start Time), and LFT (Late Finish Time) to calculate the float and slack in a project schedule.

Here are some key points to remember:

- EFT is the earliest possible finish time of an activity

- EFT is calculated by adding the duration to the EST

- EFT is used to calculate the float and slack in a project schedule

I hope that helps! Let me know if you have any other questions.

Here are the definitions and calculations for LST, LFT, and Float:

LST (Late Start Time):

The latest possible start time of an activity without delaying the project.

Calculation:

LST = LFT - Duration

Where:

- LFT (Late Finish Time) is the latest possible finish time of the activity

- Duration is the estimated time required to complete the activity

LFT (Late Finish Time):

The latest possible finish time of an activity without delaying the project.

Calculation:

LFT = EFT + Float

Where:

- EFT (Early Finish Time) is the earliest possible finish time of the activity

- Float is the amount of time an activity can be delayed without affecting the project schedule

Float:

The amount of time an activity can be delayed without affecting the project schedule.

Calculation:

Float = LFT - EFT

Where:

- LFT (Late Finish Time) is the latest possible finish time of the activity

- EFT (Early Finish Time) is the earliest possible finish time of the activity

Total Float (TF):

The total amount of float available for an activity.

Calculation:

TF = LFT - EST

Where:

- LFT (Late Finish Time) is the latest possible finish time of the activity

- EST (Early Start Time) is the earliest possible start time of the activity

Free Float (FF):

The amount of float available for an activity without affecting the early start time of its successor activities.

Calculation:

FF = min (LST (successor) - EFT (activity), TF (activity))

Where:

- LST (successor) is the late start time of the successor activity

- EFT (activity) is the early finish time of the activity

- TF (activity) is the total float of the activity

I hope this helps clarify the calculations for LST, LFT, and Float! Let me know if you have any further questions.

Probability considerations in PERT (Program Evaluation and Review Technique) are used to account for uncertainty in activity durations and project completion times. Here are some key probability considerations in PERT:

1. Activity duration uncertainty: PERT uses probabilistic estimates for activity durations, such as optimistic, most likely, and pessimistic times.

2. Probability distributions: PERT assumes that activity durations follow a beta distribution, which is a continuous probability distribution defined by two shape parameters.

3. Expected value: The expected value of an activity duration is calculated using the formula: (Optimistic + 4 x Most Likely + Pessimistic) / 6

4. Variance: The variance of an activity duration is calculated using the formula: ((Pessimistic - Optimistic) / 6)^2

5. Standard deviation: The standard deviation of an activity duration is the square root of the variance.

6. Probability of completion: PERT calculates the probability of completing the project by a given date using the cumulative distribution function (CDF) of the project completion time.

7. Sensitivity analysis: PERT performs sensitivity analysis to identify critical activities and assess the impact of changes in activity durations on the project completion time.

Time-Cost Tradeoff (TCT) is a concept in project management that refers to the relationship between the duration of a project and its cost. It suggests that there is a tradeoff between the two, meaning that reducing the duration of a project will typically increase its cost, and vice versa.

The TCT concept is often represented graphically, with time on the x-axis and cost on the y-axis. The curve typically shows that as the project duration decreases, the cost increases exponentially.

There are several strategies for managing the time-cost tradeoff, including:

1. Crashing: Reducing the project duration by adding more resources, which increases cost.

2. Fast tracking: Overlapping activities to reduce duration, which may increase cost.

3. Schedule compression: Reducing duration by reducing scope or extending working hours.

By understanding the time-cost tradeoff, project managers can make informed decisions about how to allocate resources and manage the project scope, schedule, and budget to achieve the desired outcomes.

Here's a simple example:

Project Duration (months)	Cost ($)
6 months	$100,000
5 months	$120,000
4 months	$150,000
3 months	$200,000

In this example, reducing the project duration by 1 month increases the cost by $20,000. This illustrates the time-cost tradeoff, where shorter duration results in higher cost.

8. Risk analysis: PERT uses risk analysis to identify potential risks and their impact on the project completion time.

By considering these probability factors, PERT provides a more realistic and robust project schedule, allowing for better risk management and decision-making.